RISE:2018

RESEARCH, INNOVATION AND SCHOLARSHIP EXPO

Graduate
Category: Computer and Information Sciences
Degree Seeking: MS in Electrical and Computer Engineering
Abstract ID# 2078

Northeastern University

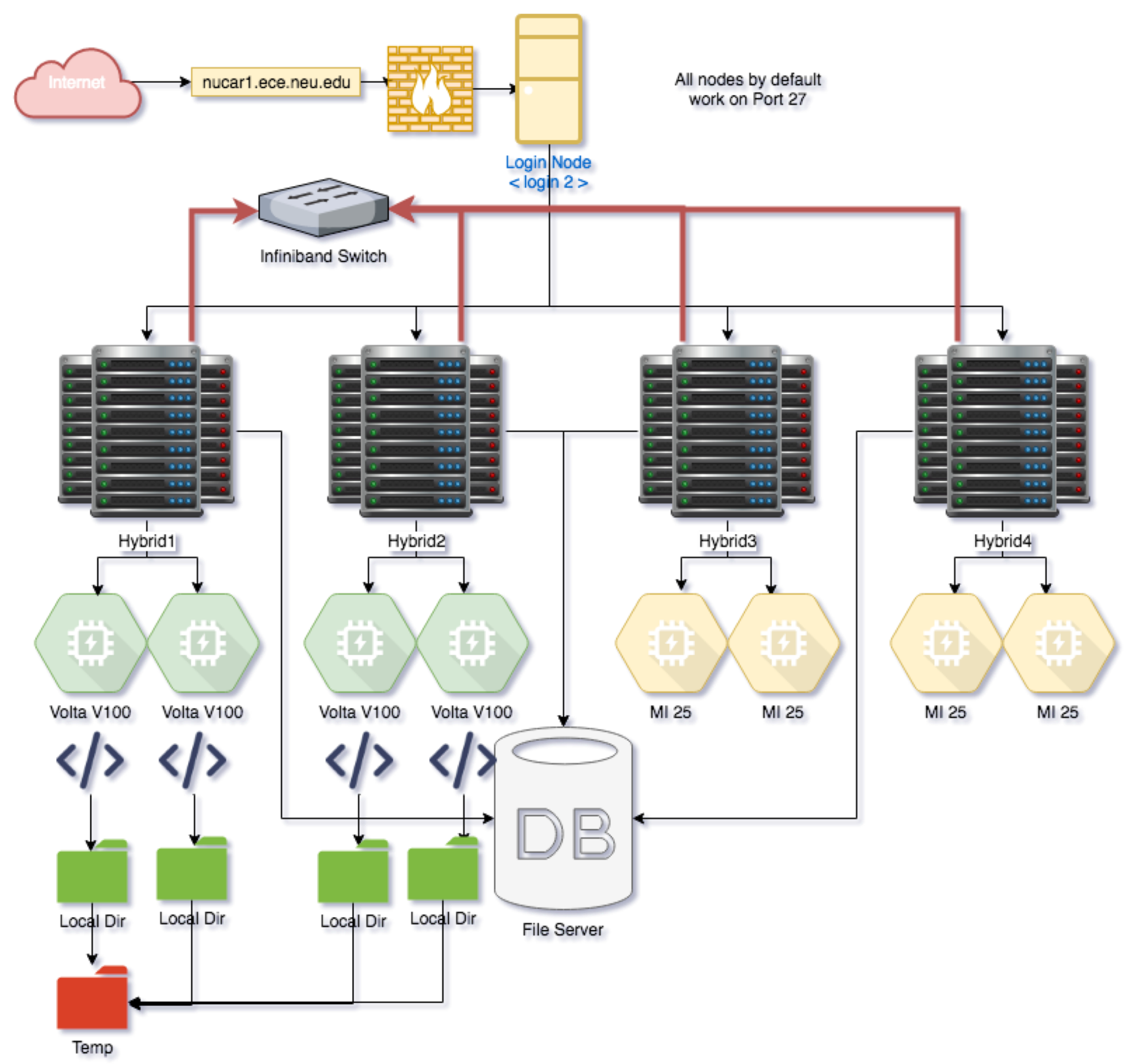
The Prime Hexagon

Kaustubh Shivdikar, Julian Gutierrez, Nicholas Velcea, Samuel Smucny, Morgan Rosenkranz, David Kaeli

Opportunity

Abstract

- Distribution of integer prime numbers remains a mystery till date.
- There has been no mathematical equation derived that explains which numbers will be prime.
- We leverage a novel approach to visualize prime numbers using overlapping hexagons that can bound all prime numbers.



Background

- In our previous work [1], we worked with a single node implementation for primes calculation with single thread.
- Our current library is capable of running over 32 GPUs per node with 4000 cores each.

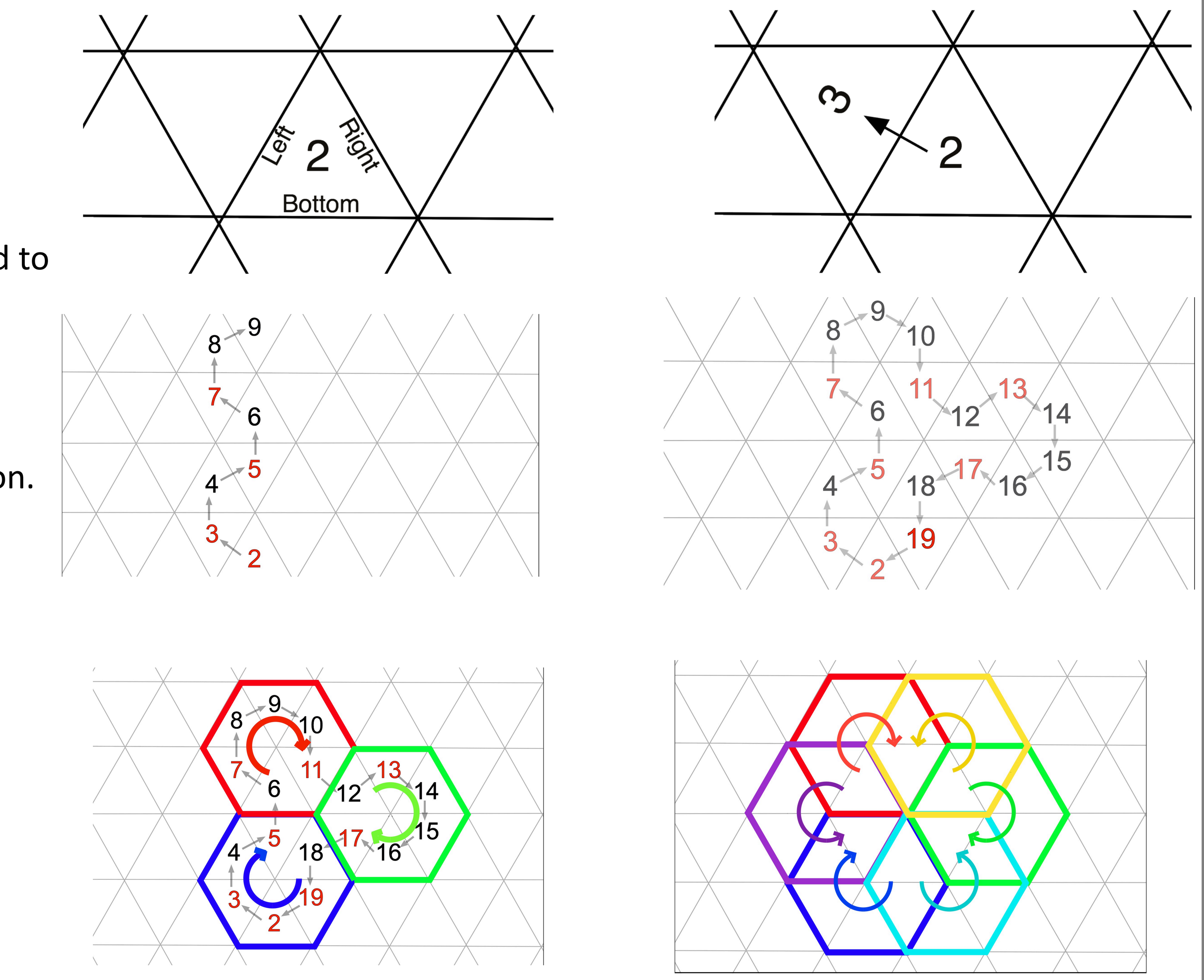
Goal

- Calculating Primes is a compute intensive process with very less scope of parallelization.
- We aim to reduce the time required to compute large primes and aim to do this using least resources (in terms of memory as well as computation) as possible.
- We achieve this by dissecting the workload and carrying out speculative coloring of primes.

Approach

Method

- A Prime Hexagon is formed when integers are sequentially added to a field of tessellating equilateral triangles.
- The path of the integers is changed whenever a prime number is encountered.
- Since prime numbers are never multiples of two or three, all numbers from "2" to infinity are confined within a 24-cell hexagon.
- Each of these six hexagons is color coded to help understand the patterns when mapped to numbers.



Implementation

- Workload is dissected into smaller batches of 10^9 numbers by master node.
- CUDA based program is distributed across multiple GPU nodes, these sieve for primes and write relative location values for every $10^{9^{th}}$ number to a 100-line file.
- Speculated position is converted to actual position by shifting all numbers of that batch.

Results

- The data below shows colors in the prime hexagon for powers of 2, 3 and truncated powers of π.
- Powers of two and three exhibit some patterns by having pairs of colors intermittently.
- A unique property observed in powers of π was that no color occurred twice consecutively.
- This has less than 1/400 chance of occurring randomly.

- The use of our cluster meant we could parallelize the processes of both calculating prime numbers and coloring them to be stored in our database.
- We used four NVIDIA V100 GPUs and four AMD EPYC 7551 CPUs and four MI 25s, totaling over 20,480 CUDA cores and 512 CPU cores, achieving a 9X speedup versus serial computation.

2 and its powers

n	a(n)	Spin	Polarity
1	2	Blue	>>>>
2	4	Blue	>>>>
3	8	Red	>>>>
4	16	Green	>>>>
5	32	Cyan	<<<<<
6	64	Green	>>>>
7	128	Purple	<<<<<
8	256	Green	>>>>
9	512	Cyan	<<<<<
10	1024	Red	>>>>
11	2048	Purple	<<<<<
12	4096	Blue	>>>>
13	8192	Green	>>>>
14	16384	Green	>>>>
15	32768	Blue	>>>>
16	65536	Green	>>>>
17	131072	Yellow	<<<<<
18	262144	Green	>>>>
19	524288	Red	>>>>
20	1048576	Cyan	<<<<<
21	2097152	Cyan	<<<<<
22	4194304	Purple	<<<<<
23	8388608	Purple	<<<<<
24	16777216	Yellow	<<<<<
25	33554432	Purple	<<<<<
26	67108864	Purple	<<<<<
27	134217728	Yellow	<<<<<
28	268435456	Cyan	<<<<<

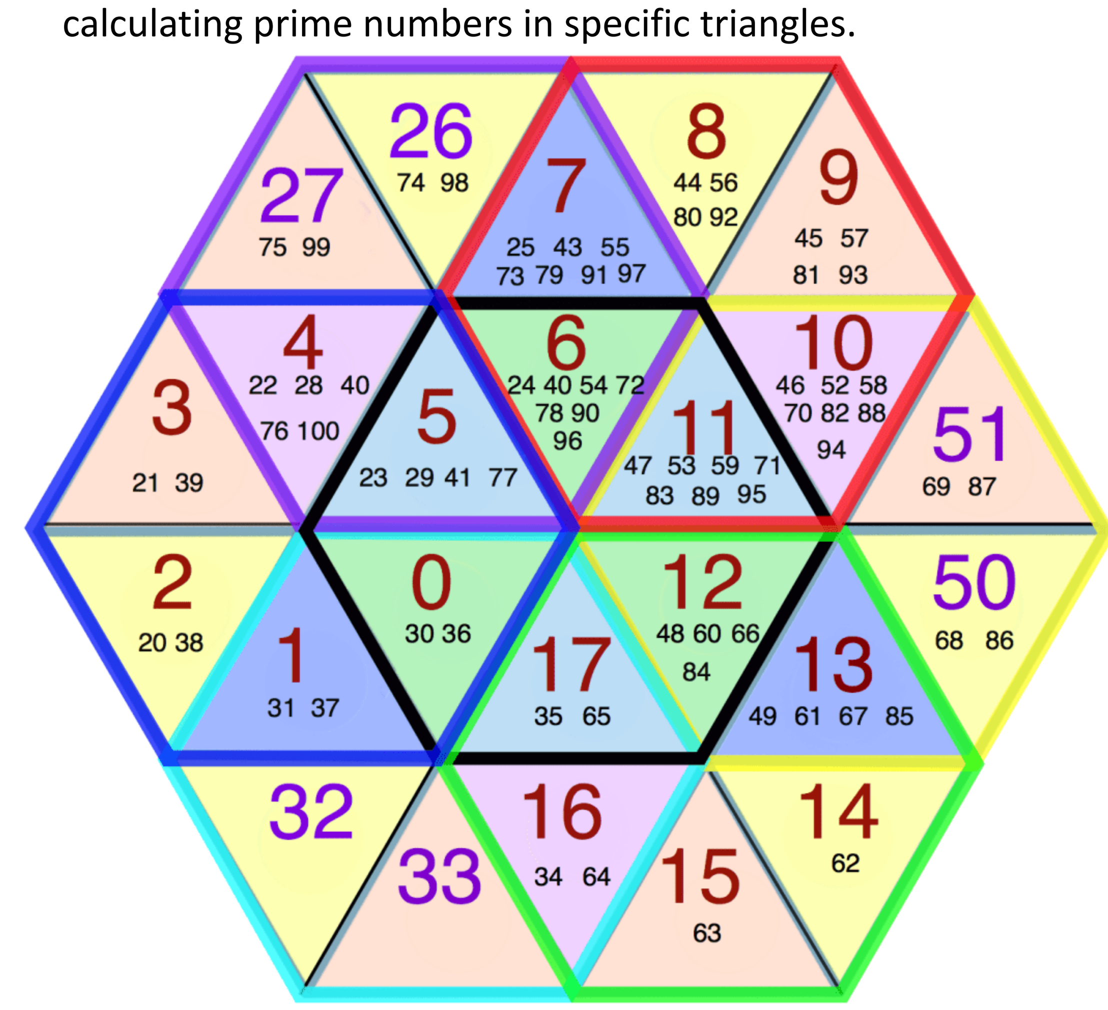
Powers of 3

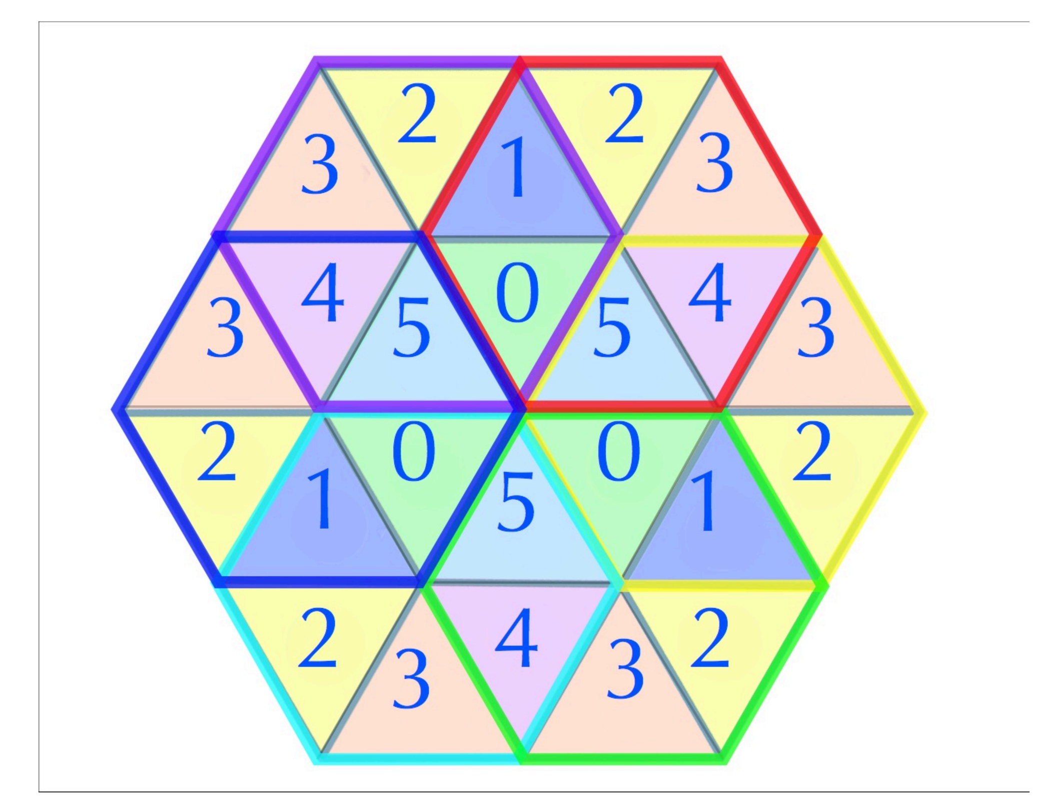
n	Number	Spin	Polarity
1	3	Blue	>>>>
2	9	Red	>>>>
3	27	Purple	<<<<<
4	81	Red	>>>>
5	243	Cyan	<<<<<
6	729	Yellow	<<<<<
7	2187	Purple	<<<<<
8	6561	Purple	<<<<<
9	19683	Cyan	<<<<<
10	59049	Blue	>>>>
11	177147	Yellow	<<<<<
12	531441	Green	>>>>
13	1594323	Purple	<<<<<
14	4782969	Yellow	<<<<<
15	14348907	Green	>>>>
16	43046721	Cyan	<<<<<
17	129140163	Cyan	<<<<<
18	387420489	Blue	>>>>
19	1162261467	Red	>>>>
20	3486784401	Blue	>>>>
21	10460353203	Cyan	<<<<<
22	31381059609	Red	>>>>
23	94143178827	Red	>>>>
24	282429536481	Yellow	<<<<<
25	847288609443	Red	>>>>
26	2541865828329	Blue	>>>>
27	7,625,597,484,987	Green	>>>>
28	22,876,792,454,961	Green	>>>>

n	Truncated Powers of Pi	Spin	Polarity
2	3	Blue	>>>>>>
3	9	Red	>>>>>>
4	31	Cyan	<<<<<<
5	97	Purple	<<<<<<
6	306	Red	>>>>>>
7	961	Green	>>>>>>
8	3,020	Cyan	<<<<<<
9	9,488	Purple	<<<<<<
10	29,809	Yellow	<<<<<<
11	93,648	Green	>>>>>>
12	294,204	Purple	<<<<<<
13	924,269	Red	>>>>>>
14	2,903,677	Blue	>>>>>>
15	9,122,171	Purple	<<<<<<
16	28,658,145	Red	>>>>>>
17	90,032,220	Yellow	<<<<<<
18	282,844,563	Red	>>>>>>
19	888,582,403	Cyan	<<<<<<
20	2,791,563,949	Yellow	<<<<<<
21	8,769,956,796	Red	>>>>>>
22	27,551,631,842	Yellow	<<<<<<
23	86,556,004,191	Cyan	<<<<<<
24	271,923,706,893	Yellow	<<<<<<
25	854,273,519,913	Purple	<<<<<<
26	2,683,779,414,317	Red	>>>>>>
27	8,431,341,691,876	Purple	<<<<<<
28	26,487,841,119,103	Green	>>>>>>
29	83,214,007,069,229	Blue	>>>>>>
30	261,424,513,284,460	Green	>>>>>>
31	821,289,330,402,749	Cyan	<<<<<<
32	2,580,156,526,864,958	Yellow	<<<<<<
33	8,105,800,789,910,709	Green	>>>>>>
34	25,465,124,213,045,828	Blue	>>>>>>
35	80,001,047,150,456,339		
36	251,330,702,007,364,298		

Impact

- If we write the remainder after dividing every number by 6, every triangle will have the same remainder for all numbers, as seen in the figure (right).
- This we employ for gaining speedup in calculations by not calculating prime numbers in specific triangles.





- We are building a database of large Prime Numbers with their associated Hex color.
- Such a database of large prime numbers when made available to everyone, will allow other researchers to expand on their work.
- Many modern encryption algorithms depend on the factorization of very large primes which are hard to find.
- Recognizing patterns in Prime numbers will affect such encryption technologies.

References and Acknowledgements

[1] This research was possible owing to the support provided by mathematician Tad Gallion and his work on prime numbers.
[2] "Visualizing and Predicting Prime Numbers", infosthetics.com, December 2009.
[3] "Primal Chaos, (Visualizations)",Carlos Paris, sievesofchaos.com.