Accelerating Polynomial Multiplication for Homomorphic Encryption on GPUs

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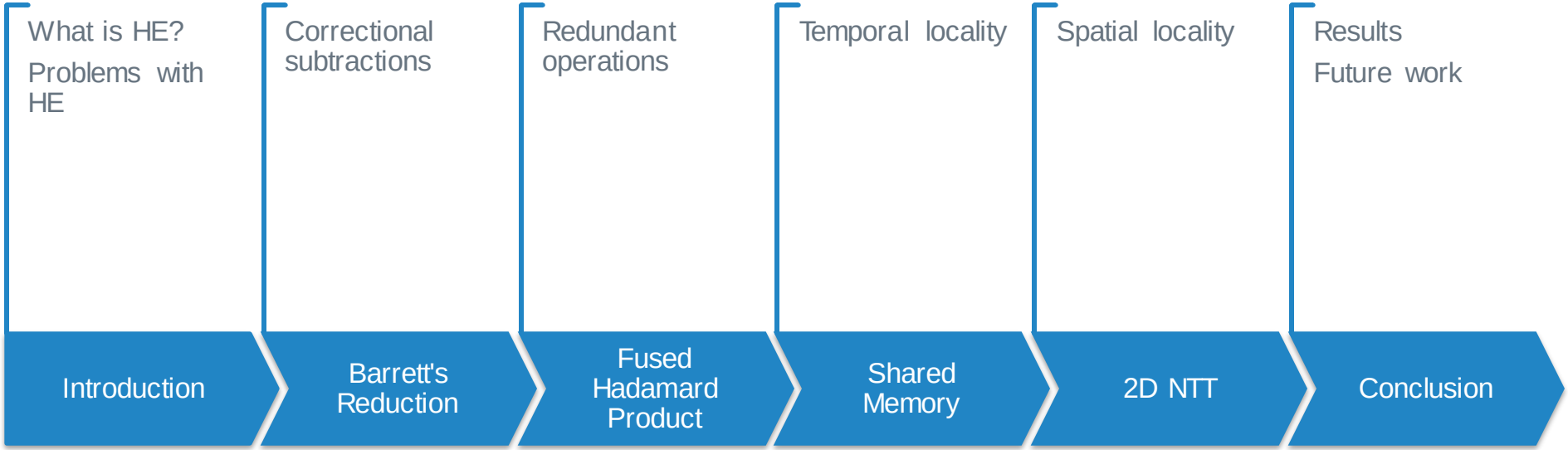
Outline


What is HE?
Problems with HE
Correctional subtractions
Redundant operations
Temporal locality
Spatial locality
Results
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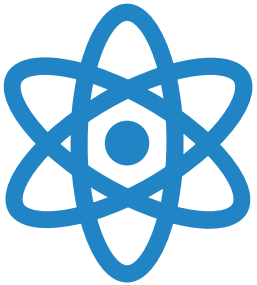
Introduction
Barrett's Reduction
Fused Hadamard Product
Shared Memory
2D NTT
Conclusion

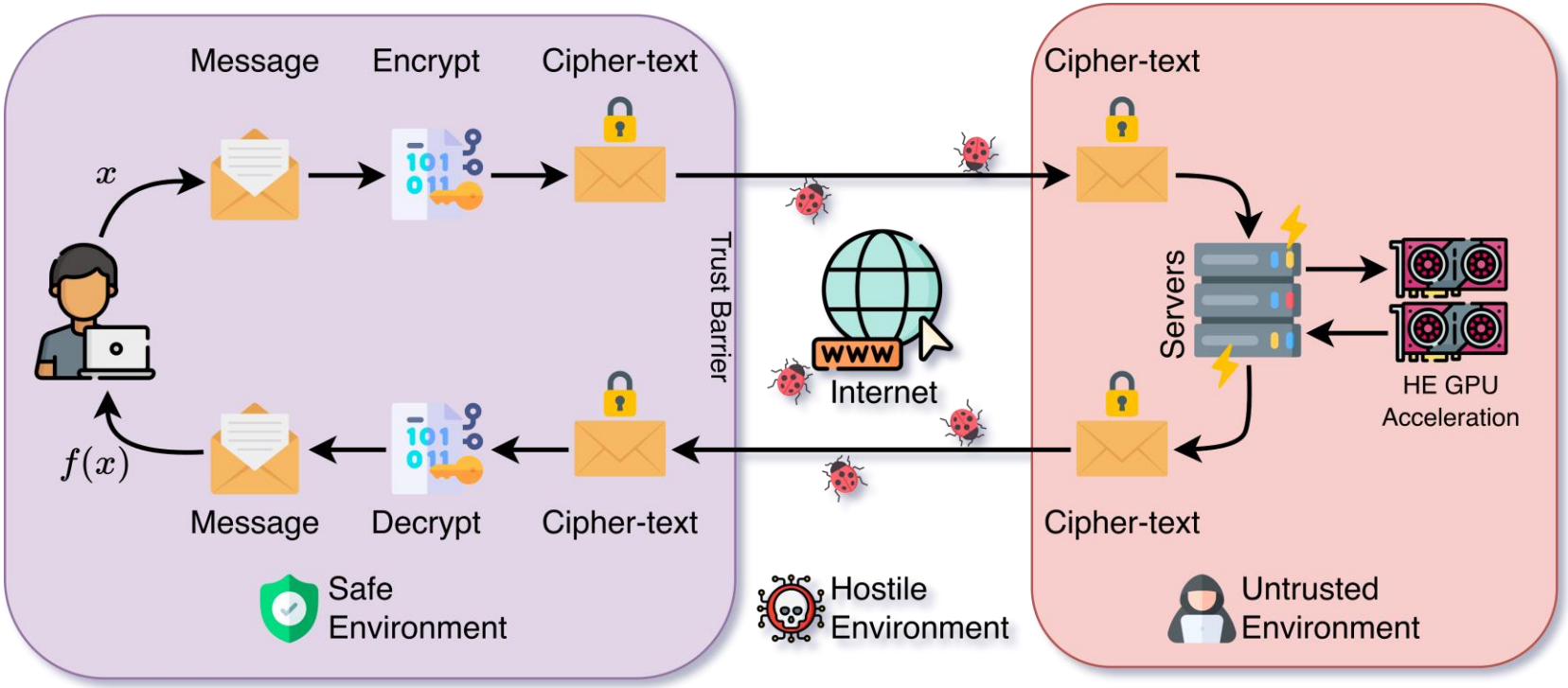
What is Homomorphic Encryption?

HE is a type of encryption that allows computation to be run on encrypted operands

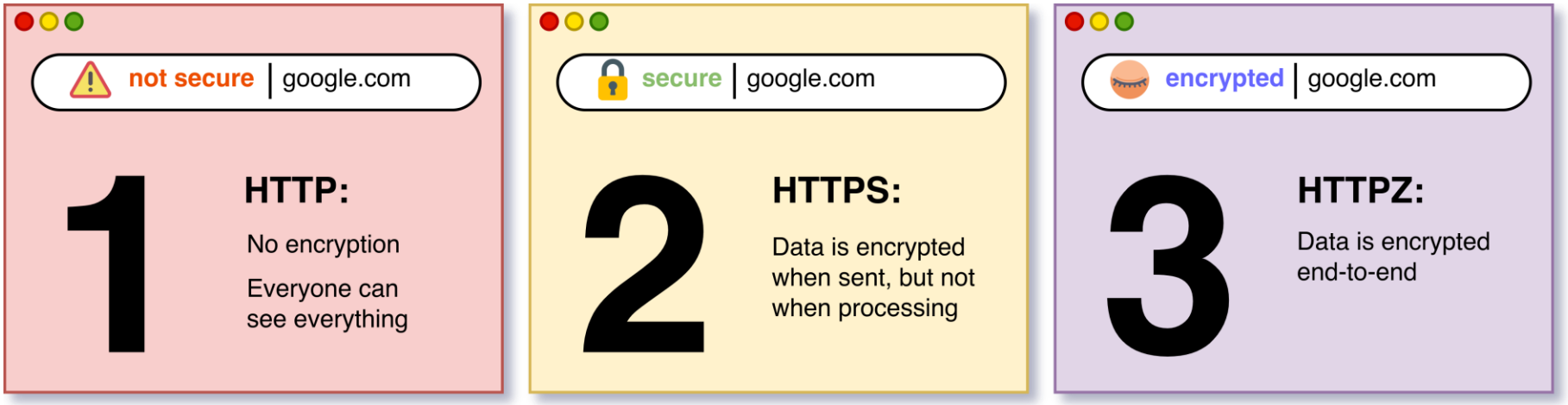


HE schemes are lattice-based, making them quantum resistant

What is Homomorphic Encryption?



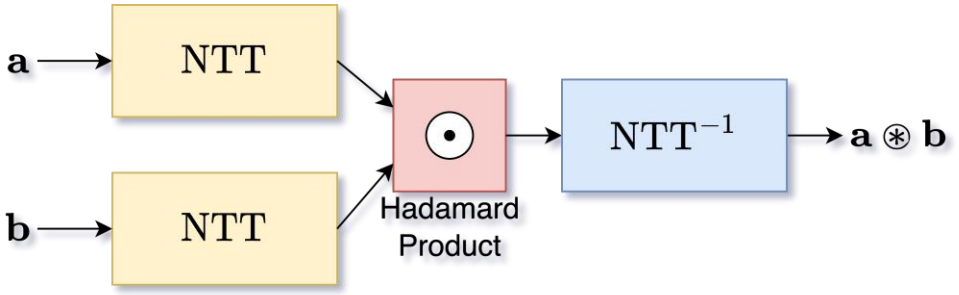
"HE could give rise to a new internet protocol, **HTTPZ**, that would standardize end-to-end encryption and replace HTTPS as the default protocol"



--- By R. Hindi, "*People shouldn't care about privacy*", ZAMA, Aug 2021

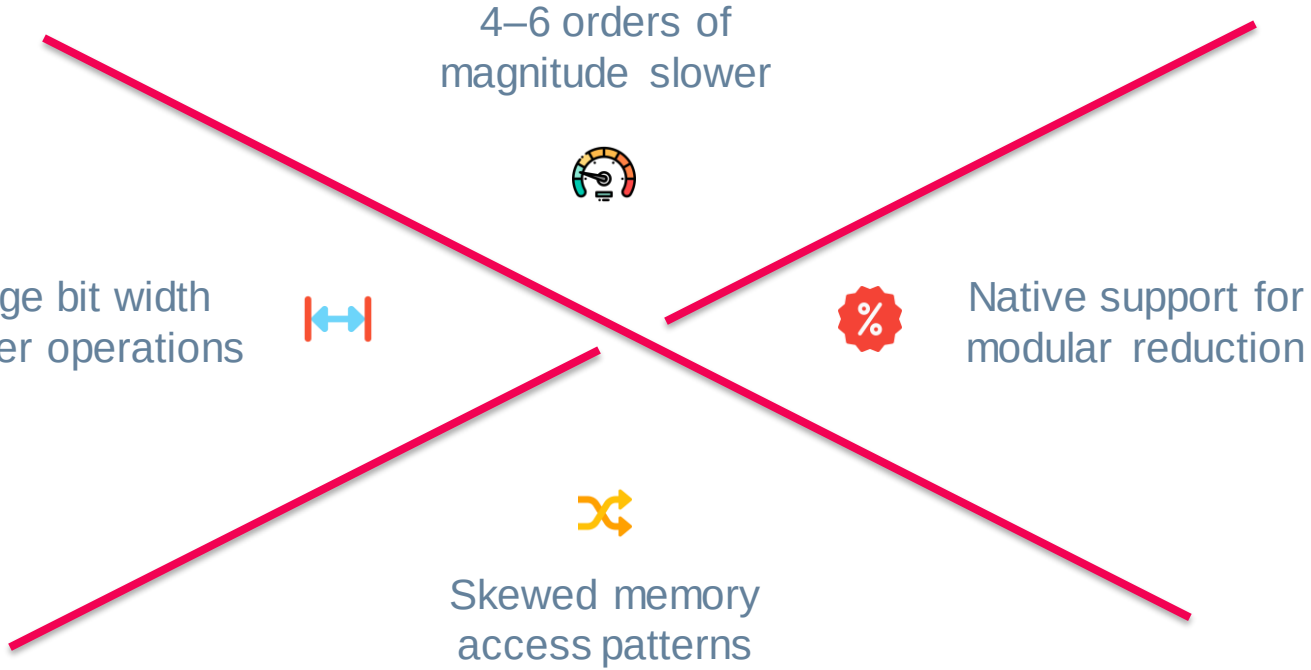
HE Bottlenecks

- **Polynomial multiplication** is the key bottleneck for lattice-based cryptography and HE
- Poly-mult is typically implemented with Number Theoretic Transform (**NTT**)
- NTT relies heavily on **modular reduction** operation



Polynomial Multiplication

Problems with HE



Contributions

Optimized Barrett's Reduction

- Fewer Correctional Subtractions

Fused Hadamard Product

- Reduces Operational Complexity

Persistent Shared Memory

- Increased Temporal Locality

Mixed Radix 2D NTT

- Increased Spatial Locality

Optimized Barrett's Reduction

- NTT is dominated by **modulo** operation
- Modulo computation involves expensive **division** operation
- **Barrett's reduction** replaces division with a set of bit-shift and multiplication operations

$$\text{Remainder} = x \,\%\, q$$

$$\text{Remainder} = x - \left(\left\lfloor \frac{x}{q} \right\rfloor \times q \right)$$

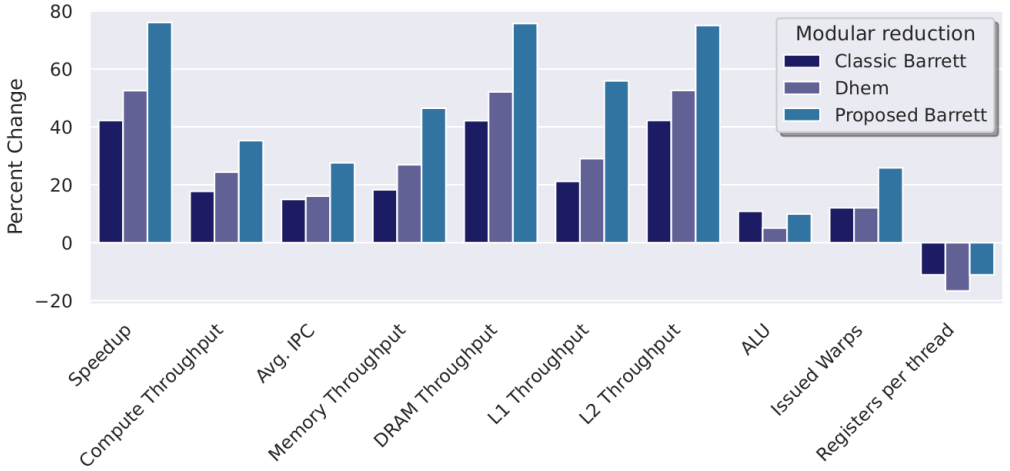
Algorithm 2 Classical Barrett reduction

Require: $m = \text{len}(q) \leq \beta - 2,\ 0 \leq x < 2^{2m},\ \mu = \lfloor \frac{2^{2m}}{q} \rfloor$
Ensure: $\text{rem} = x \bmod q$

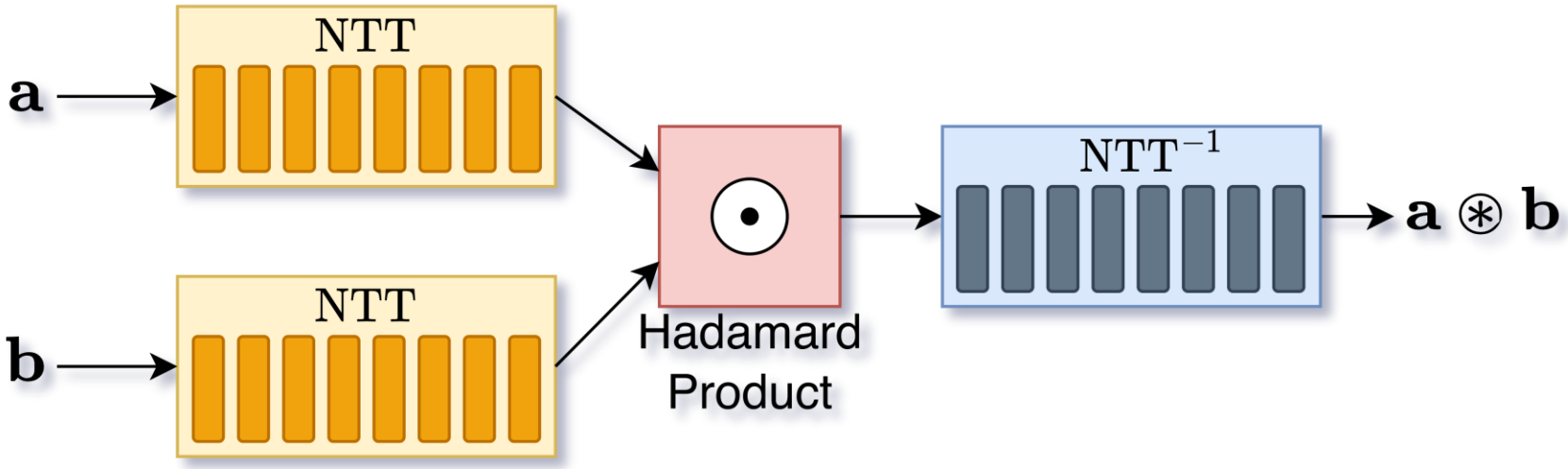
1: $c \leftarrow x \gg (m-1)$
2: $\text{quot} \leftarrow (c \times \mu) \gg (m+1)$
3: $\text{rem} \leftarrow x - \text{quot} \times q$
4: **if** $\text{rem} \geq q$ **then**
5: $\quad \text{rem} \leftarrow \text{rem} - q$
6: **if** $\text{rem} \geq q$ **then**
7: $\quad \text{rem} \leftarrow \text{rem} - q$
8: **return** rem

Optimized Barrett's Reduction

- Proposed Barrett's reduction reduces number of correctional subtractions
- Future Work: Natively supported modular reduction with hardware implementation

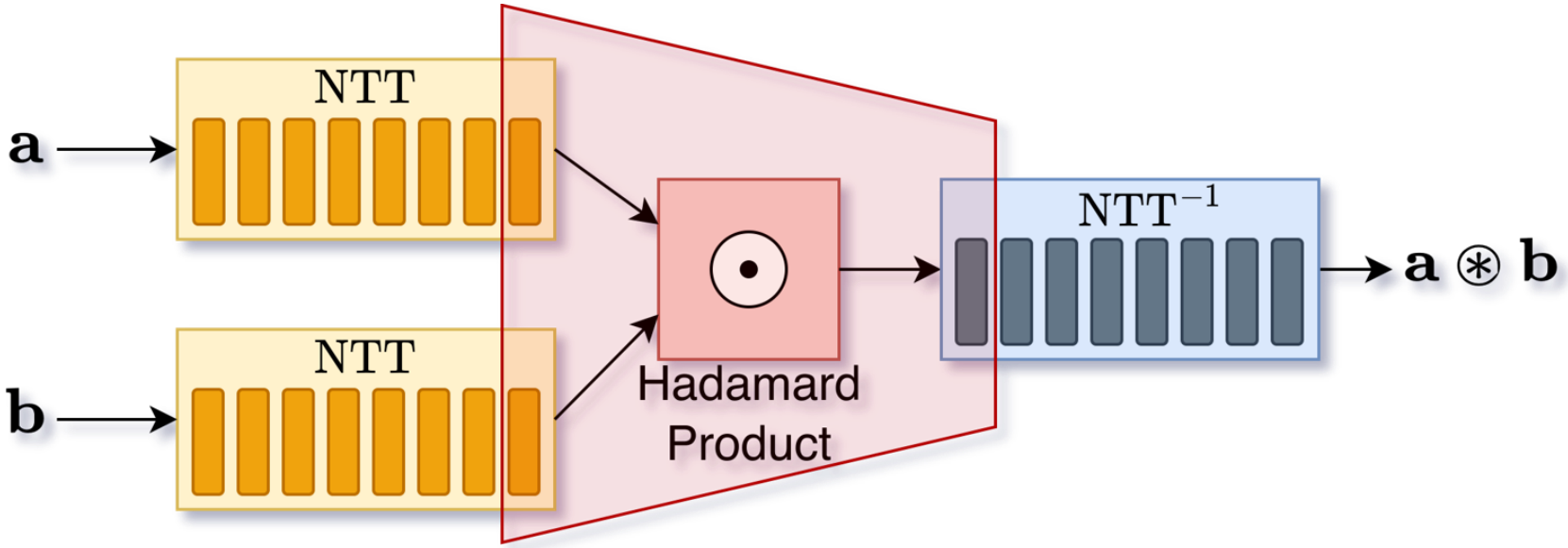


Fused Hadamard Product



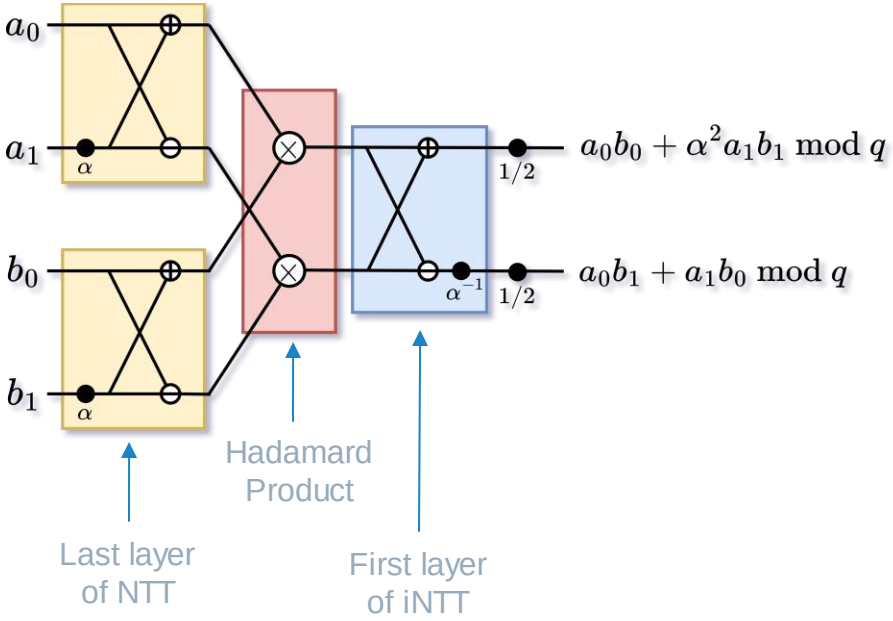
Polynomial Multiplication

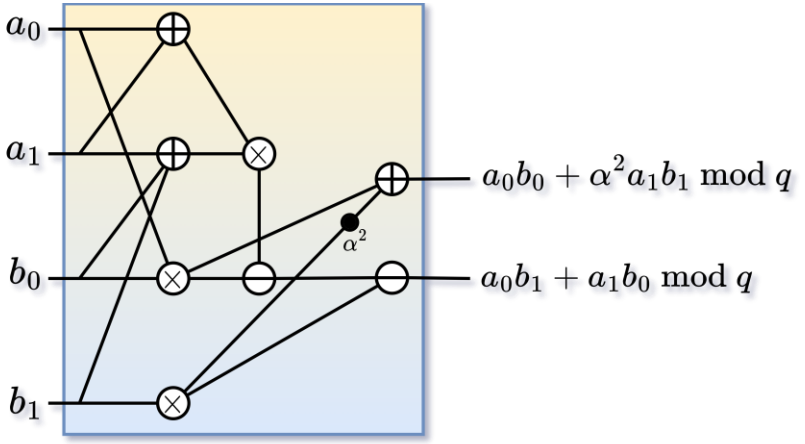
Fused Hadamard Product



Polynomial Multiplication

Fused Hadamard Product

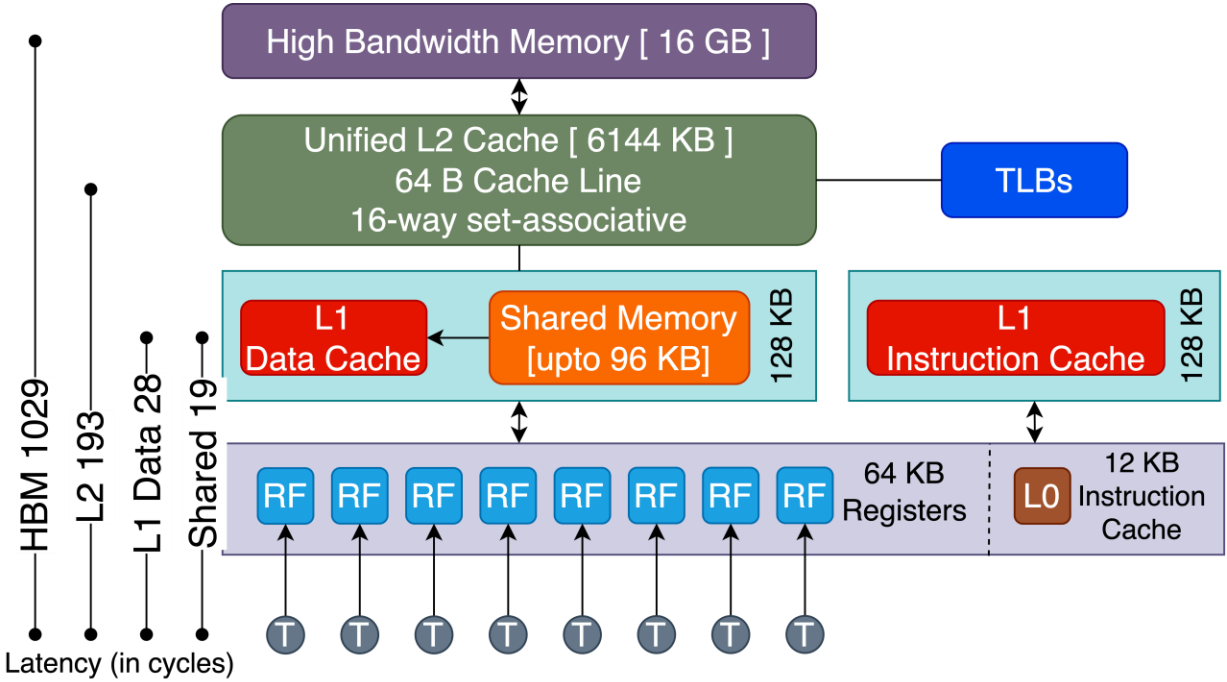




Fused NTT, iNTT, and Hadamard Product

Persistent Shared Memory

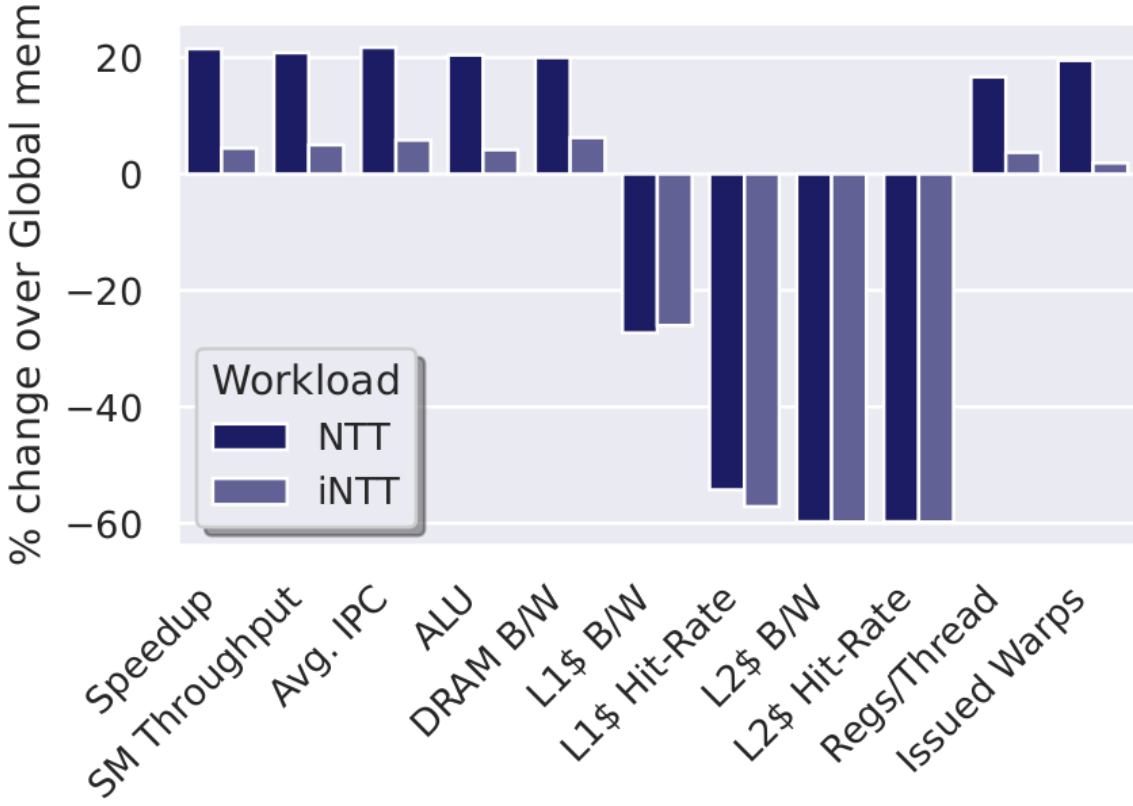
- NTT is a **memory-bound** kernel
- Each stage of NTT generates **intermediate results** for subsequent stages
- Intermediate results of each stage are **cached** on shared memory
- Removes **redundant** global memory accesses



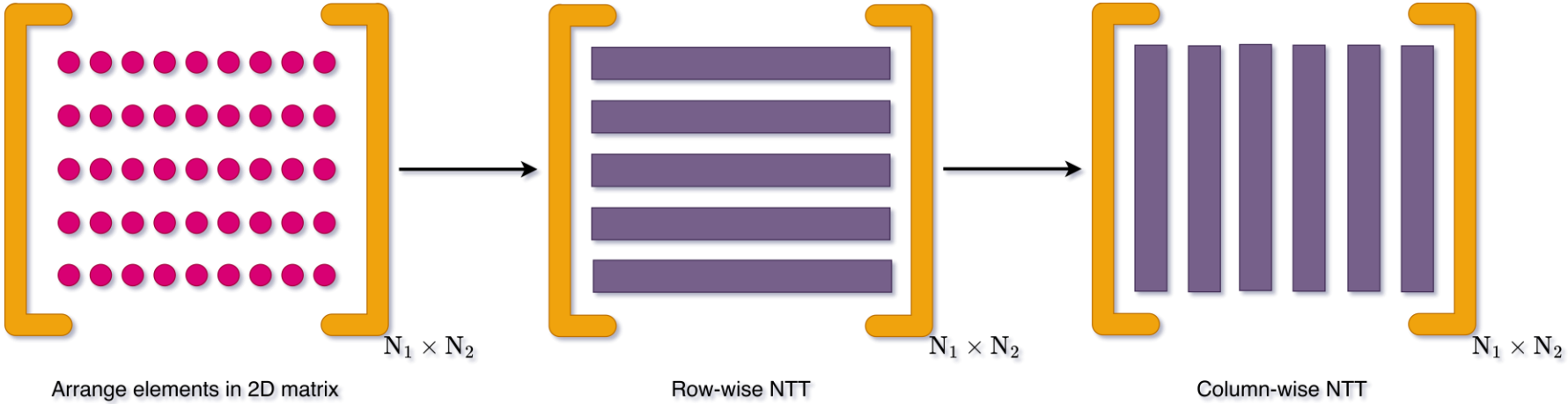
V100 GPU Memory Hierarchy

Persistent Shared Memory

- Shared memory use provides **25%** speedup over global memory
- **L1** and **L2** cache memory pressure drops significantly
- Only works for NTT sizes that fit in shared **memory size** ($N<2^{11}$)

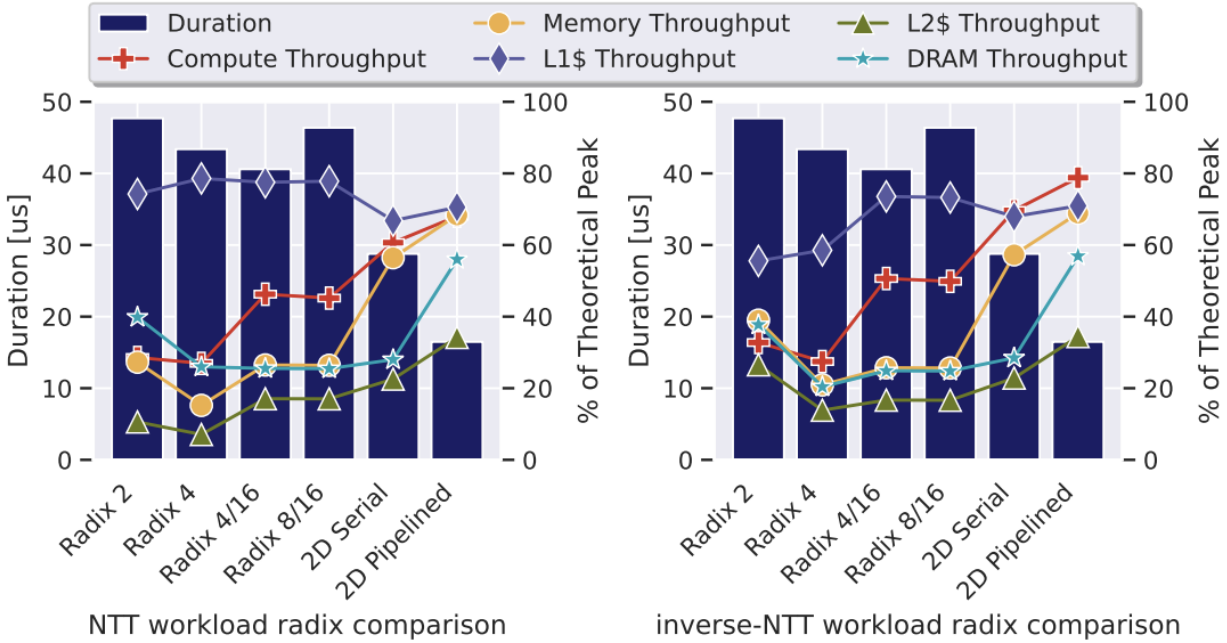


Mixed Radix 2D NTT



Mixed Radix 2D NTT

- 2D NTT increases **compute** costs
- Reduces **memory** pressure
- Preserves **spatial locality**
- Row-wise and Column-wise NTT can be **pipelined**

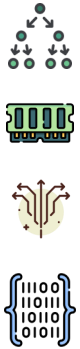


Concluding Remarks

HE is a **popular** memory-intensive workload with high computational demands

Explored key **bottlenecks** in HE

- Algorithmic improvements
- Low-level kernel improvements



Presented four optimizations

Achieved speedup

- CPU: **123.13x**
- GPU: **2.37x**

Future Work / Architectural Feature Requests

- Larger integer **bit width** support for GPU
- Native **modular reduction** support

Thank you!

Any questions?

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